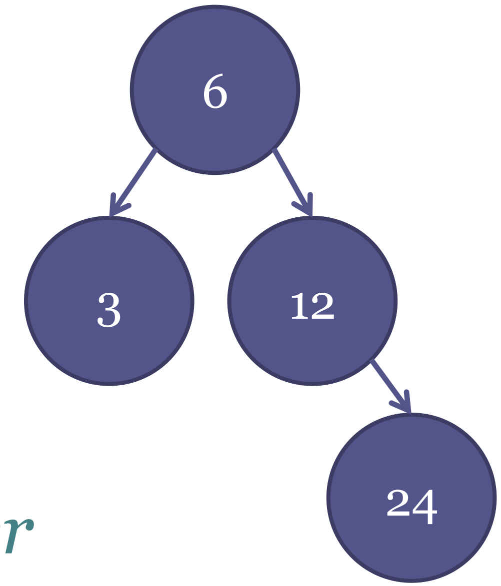
Practical Concurrent Binary Search Trees via Logical Ordering

Dana Drachsler (Technion, Israel)
Martin Vechev (ETH, Switzerland)
Eran Yahav (Technion , Israel)

Binary Search Tree

- A data-structure that stores elements
  - Each element has a unique key
  - Duplications are not allowed
- The BST consists of nodes
  - Each node stores an element
- For each node
  - Keys in the left sub-tree are *smaller*
  - Keys in the right sub-tree are *bigger*



Binary Search Tree Operations

- Contains(k): check if k is present
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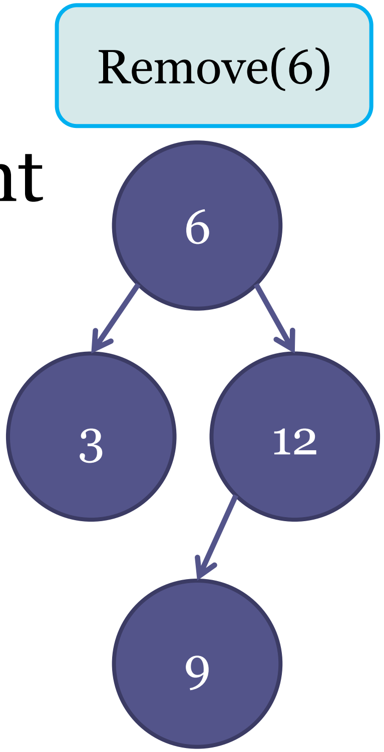
    - Find the successor: the left-most node of the right sub-tree



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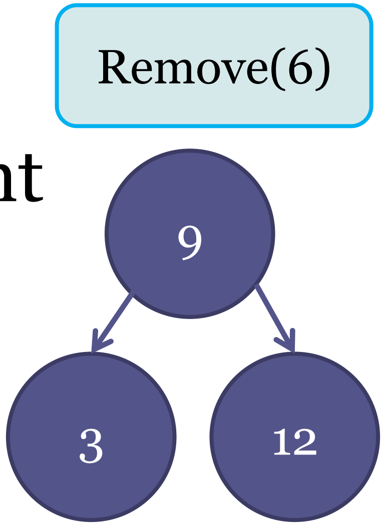
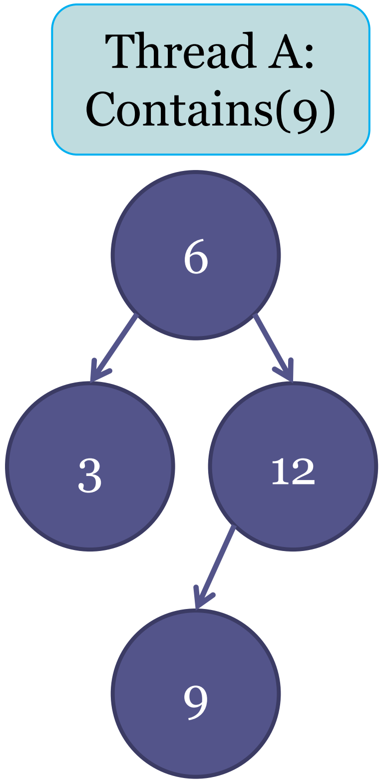
    - Relocate the successor



BST: Challenge with Concurrency

1. Thread A searches for 9



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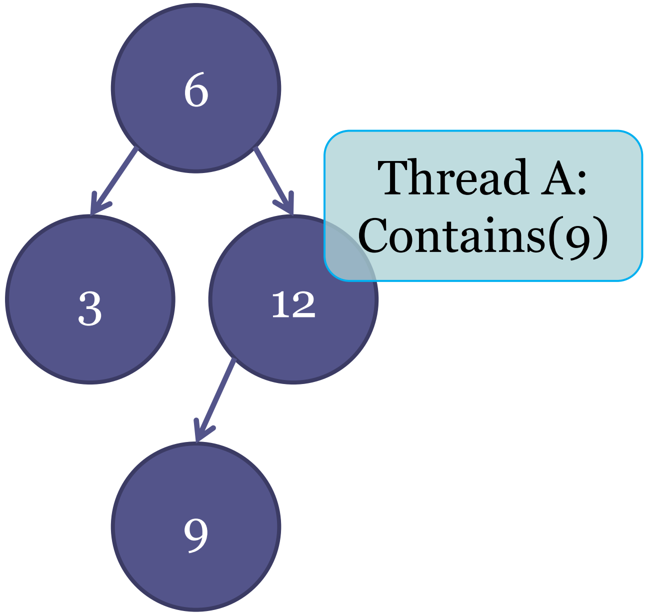

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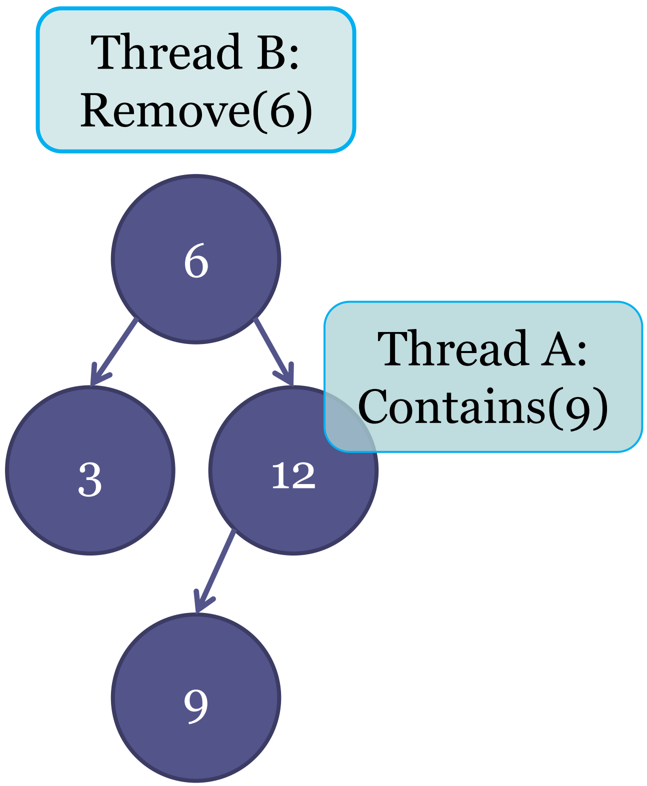

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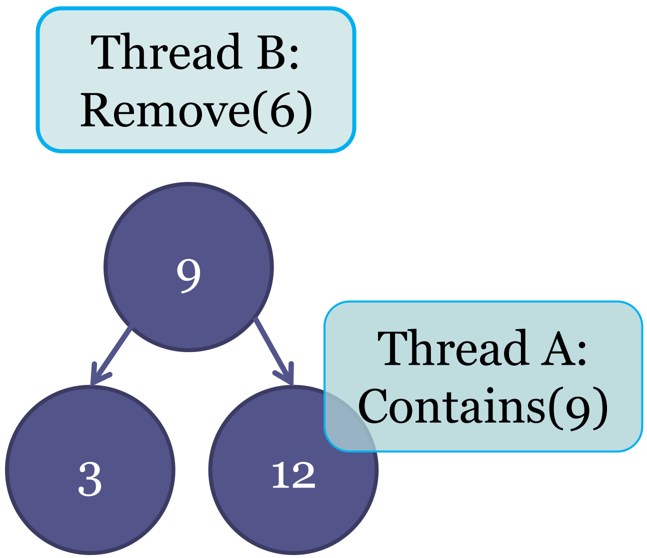

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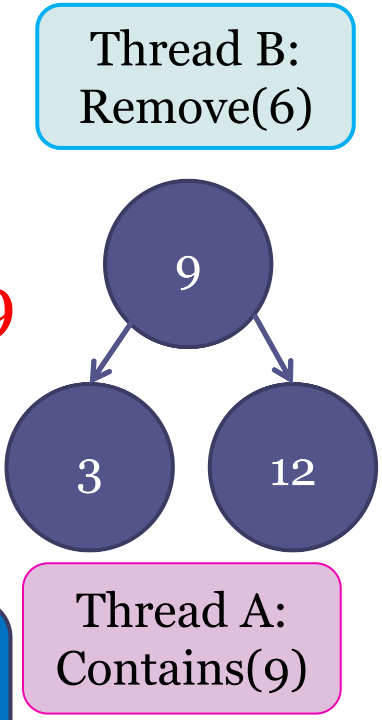


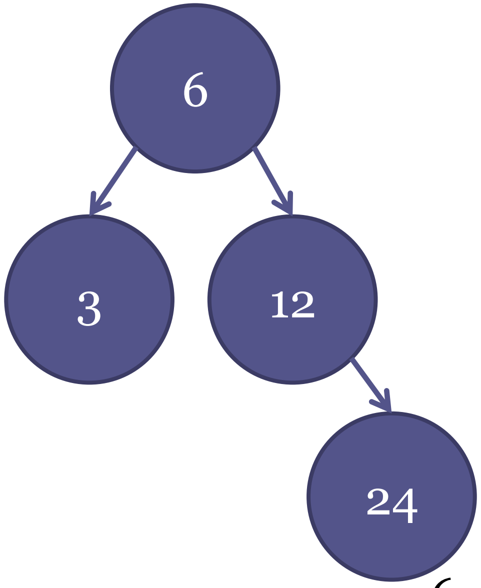
Search operation unaware of concurrent changes to BST layout

Our Contribution

- We present a new perspective on BST
  - Locking is based on a *logical ordering layout*, and not only on the BST layout
- The additional layout requires
  - Extra space for the new links
  - Extra time for maintaining the new links
  - Extra lock acquires of the new links
- Yet, it performs as state-of-the-art algorithms
  - Sometimes even better

Key Idea

- There is a total order between the keys



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Key Idea

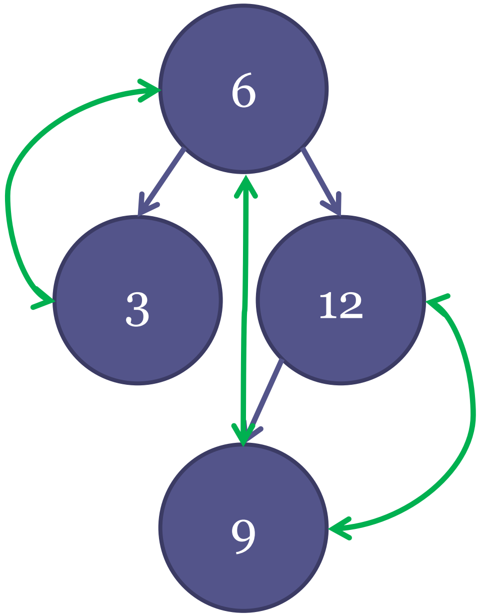
- There is a total order between the keys
- The order induces *intervals*
  - A key is present in the tree if it is an end point of some interval
- We explicitly maintain the intervals
  - *The logical ordering layout*

$-\infty < 3 < 6 < 12 < 24 < \infty$

$(-\infty, 3)$ $(3,6)$ $(6,12)$ $(12,24)$ $(24, \infty)$

Logical Ordering Layout

- Connect n to its predecessor, p, and successor, s
  - n can access efficiently to (p, n),(n, s)



Logical Ordering Layout

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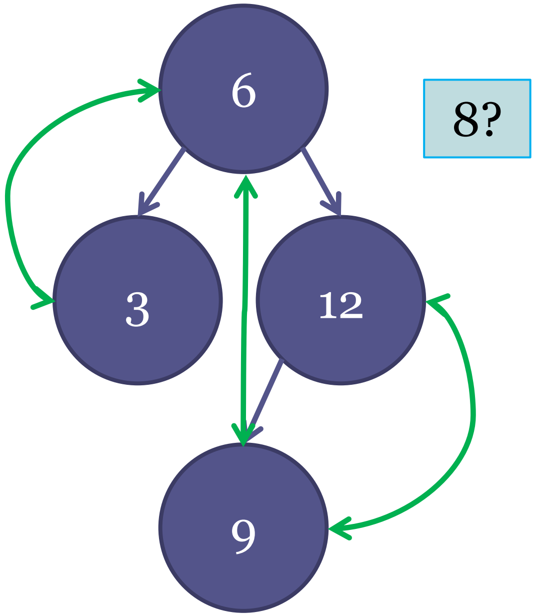

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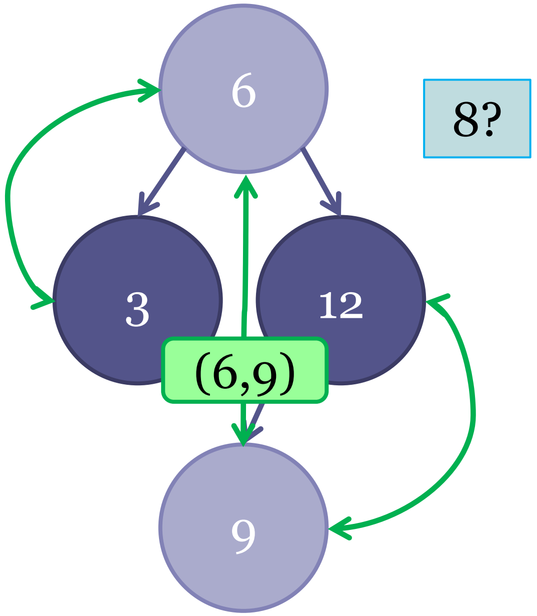
- For (p, s): p, s might be non adjacent in the tree

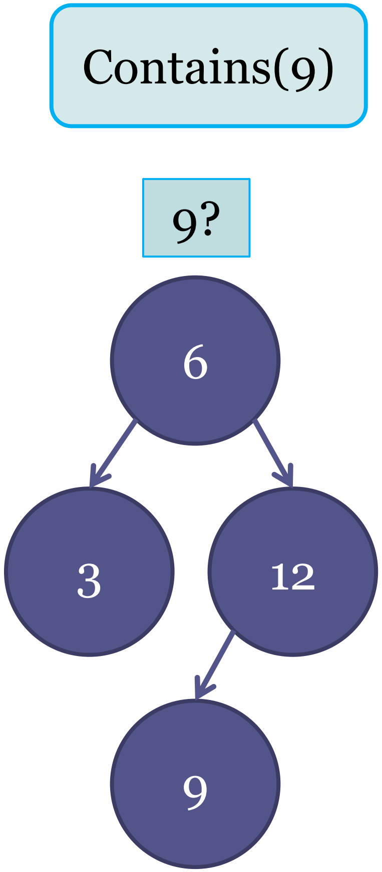


Main Advantages

- Efficiently answer membership queries even under concurrent updates to the BST layout
  - Includes relocating the successor in a removal
  - Includes applying sequential balancing operations
- Efficiently find the successor of a node
  - Important for the removal of a two-nodes parent
- Efficiently find the minimal/maximal keys
  - Can be used to implement a priority queue

The Sequential Contains(k)

- Traverse downwards in the tree



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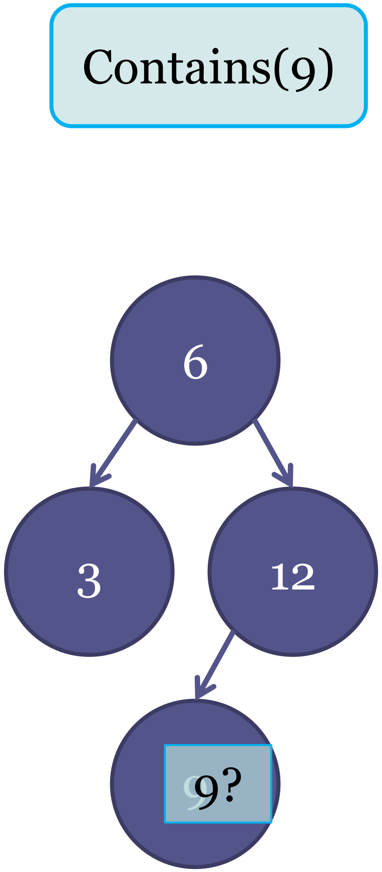

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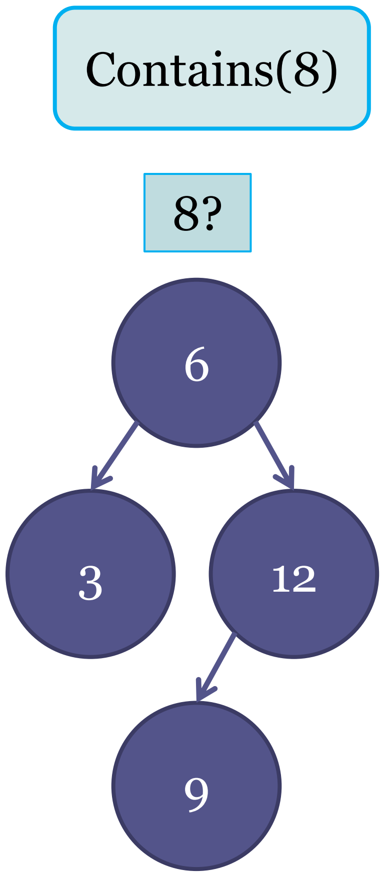

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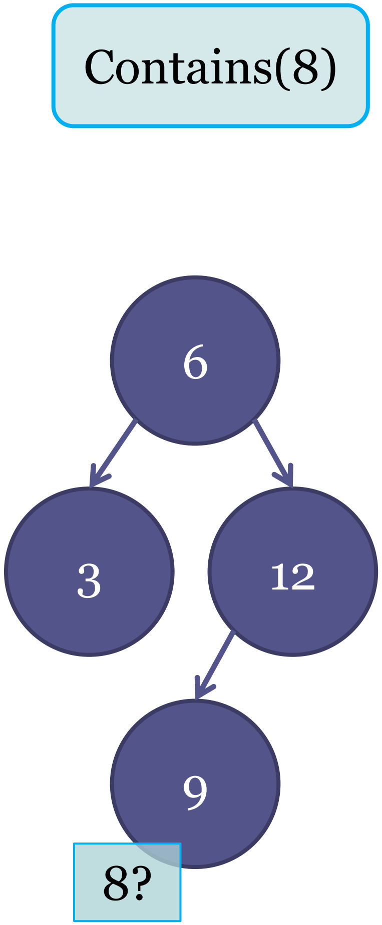

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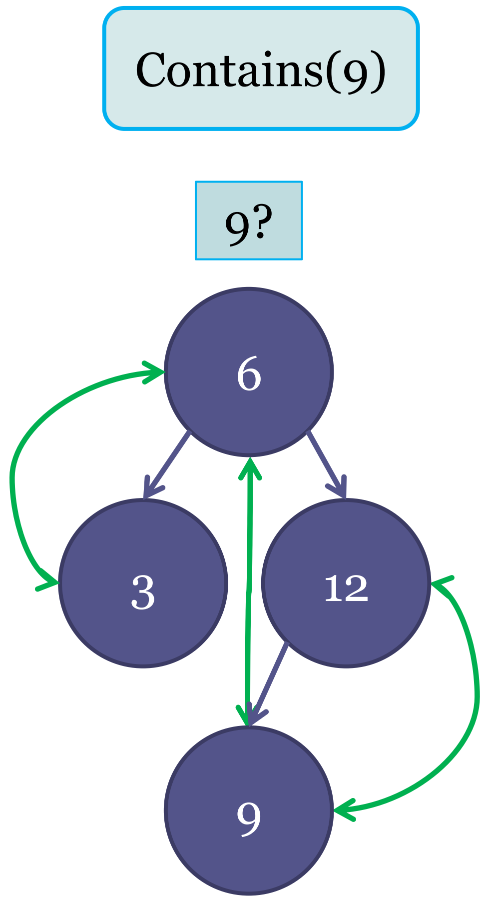

Tree link
Interval link

The Concurrent Contains(k)

Contains(9)

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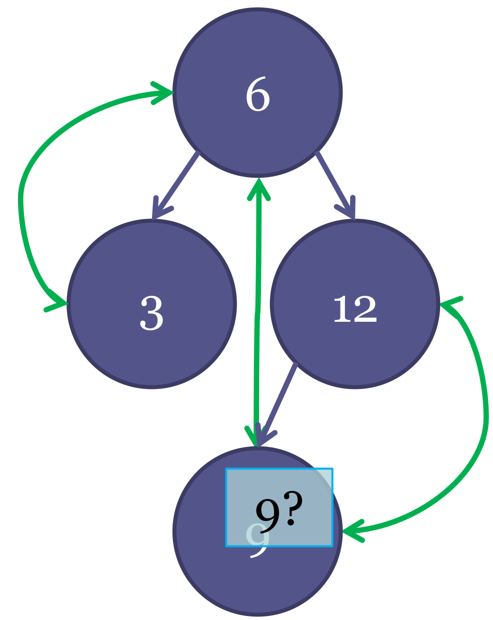

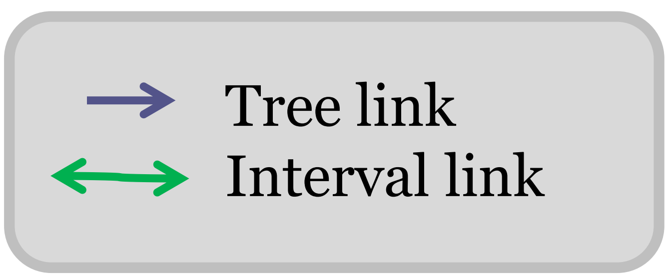

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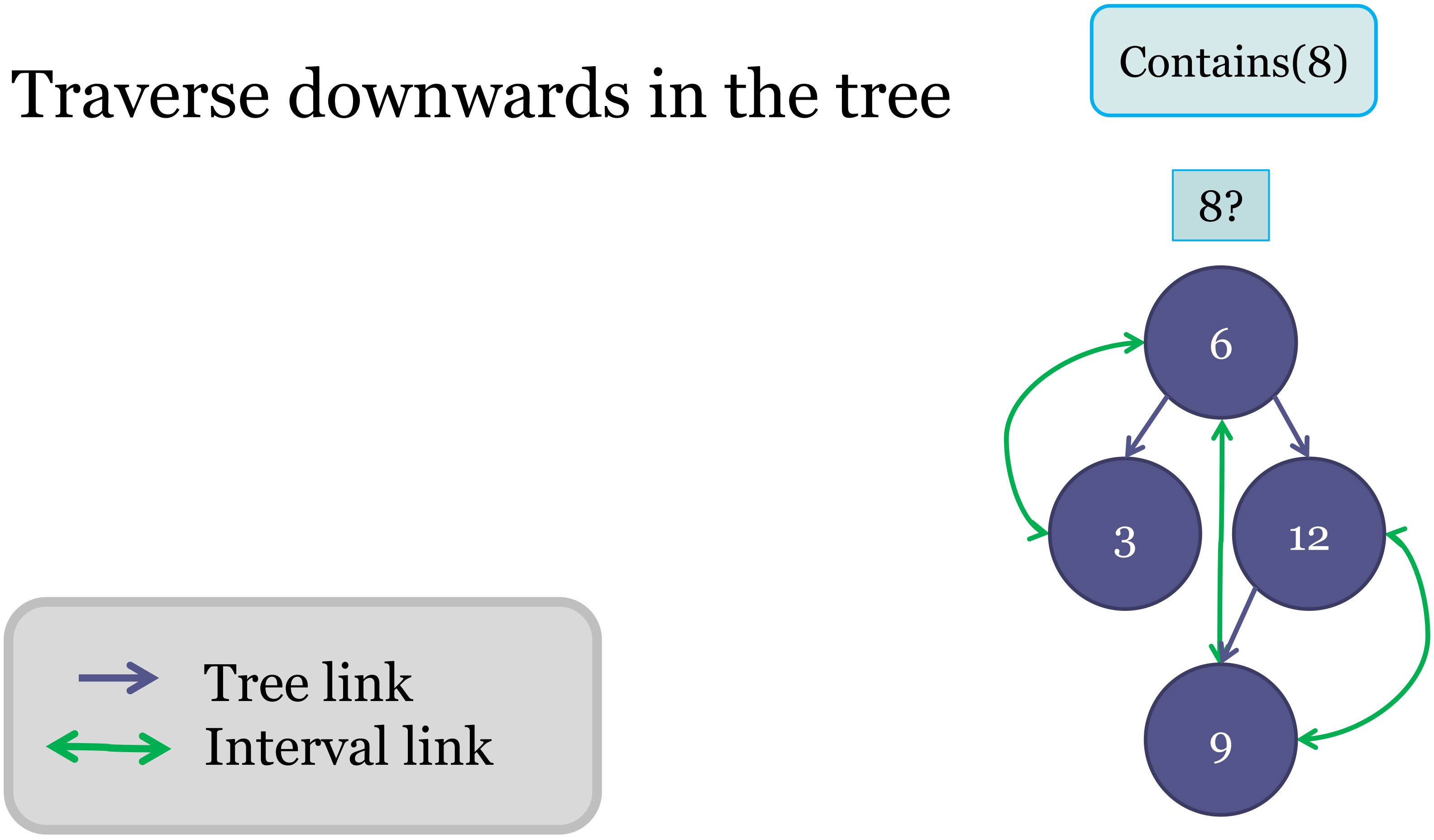
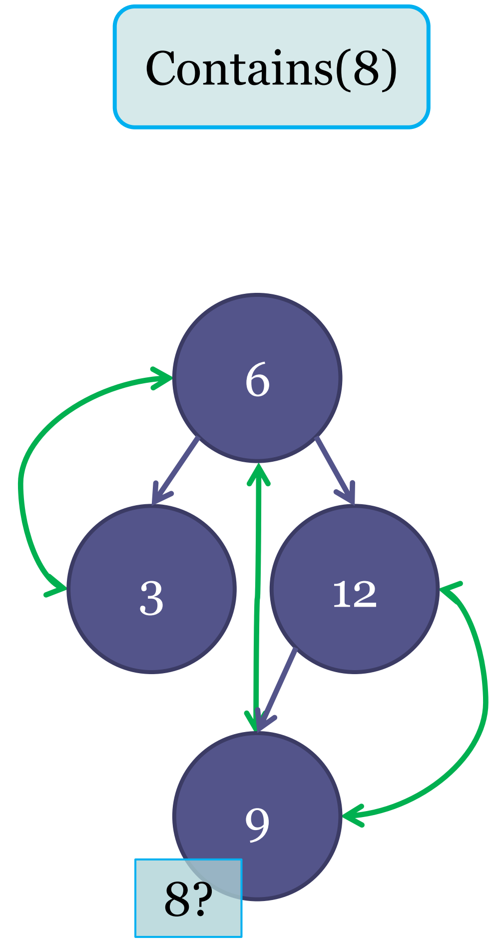


The Concurrent Contains(k)

- Traverse downwards in the tree
- If reached to an end of a path, traverse via the ordering layout to find (p, s) such that $k \in [p, s]$
  - Return false iff $k \neq p, s$



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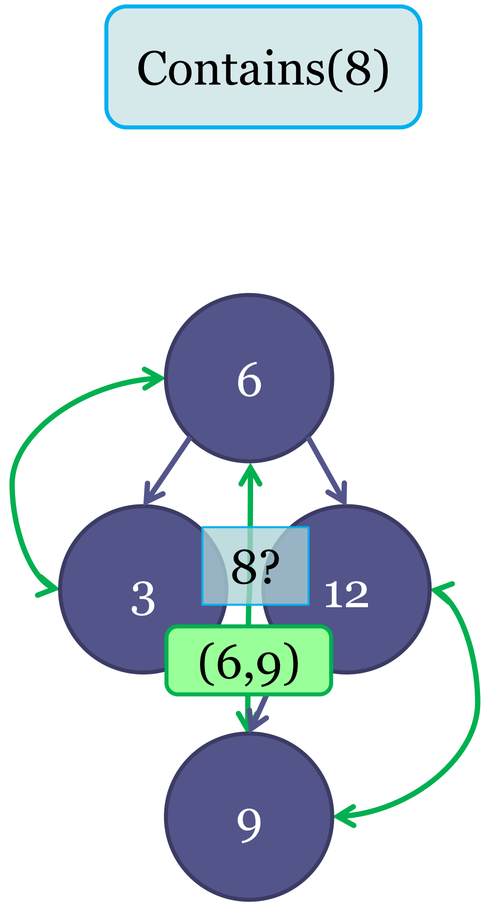
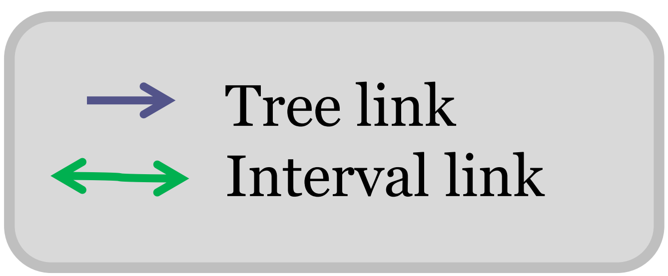
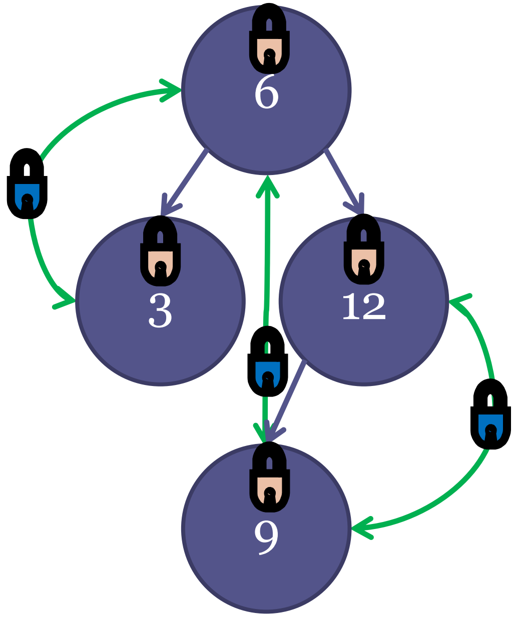
- This operation is non-blocking

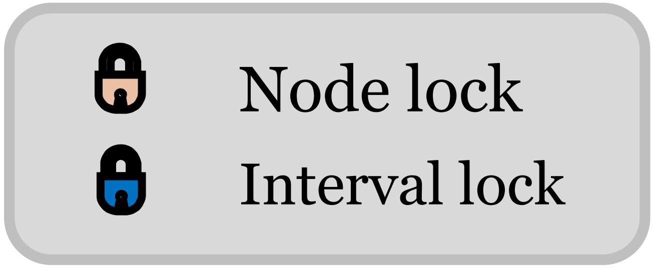




Insert and Remove Operations

- The synchronization is based on locks
- The operations lock
  - The relevant nodes in the tree
  - The relevant intervals





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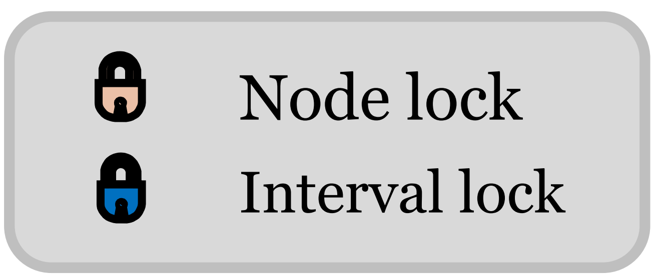

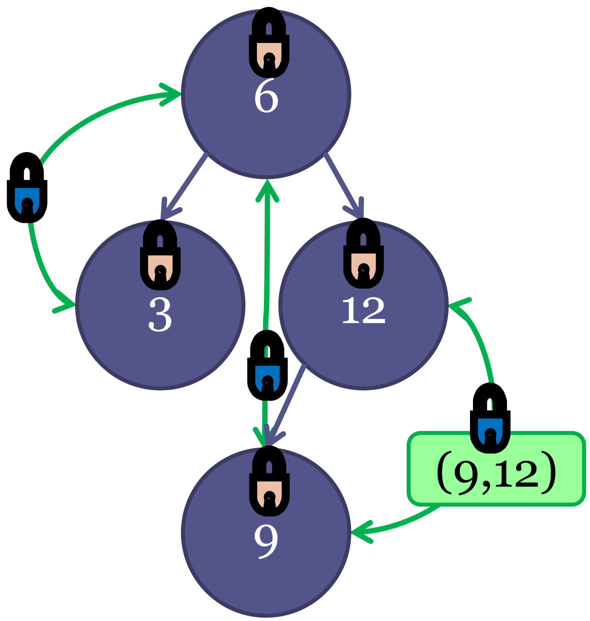

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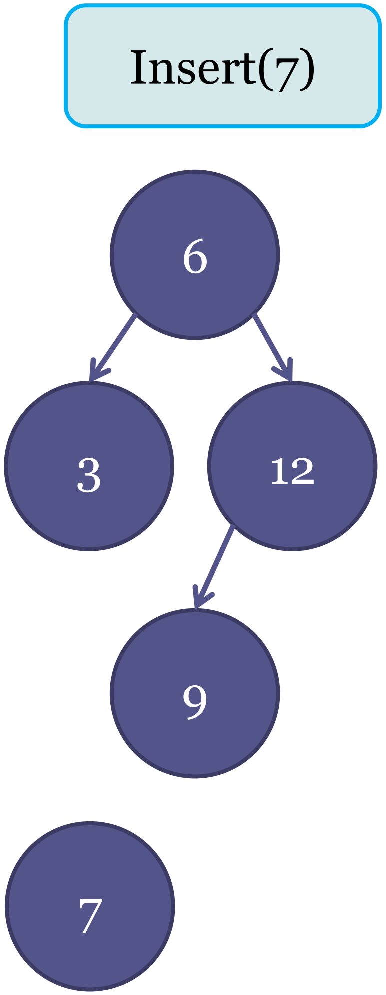

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- Traverse downwards in the tree
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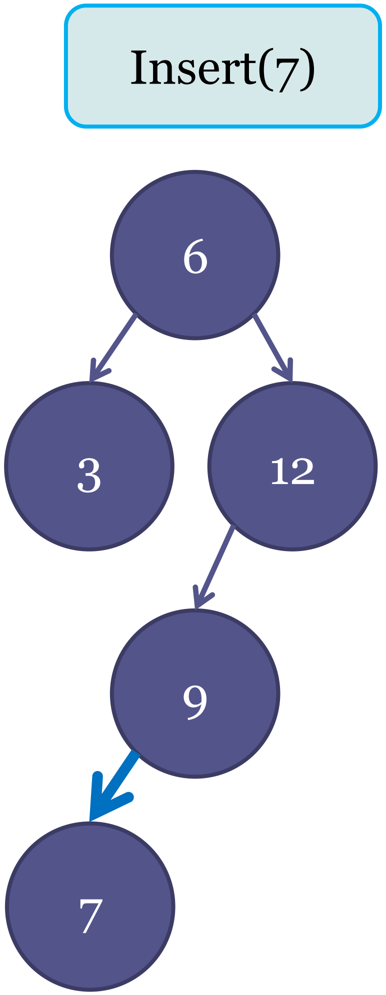
- Otherwise, let l be the node at the end of a path
  - Connect l to the new node



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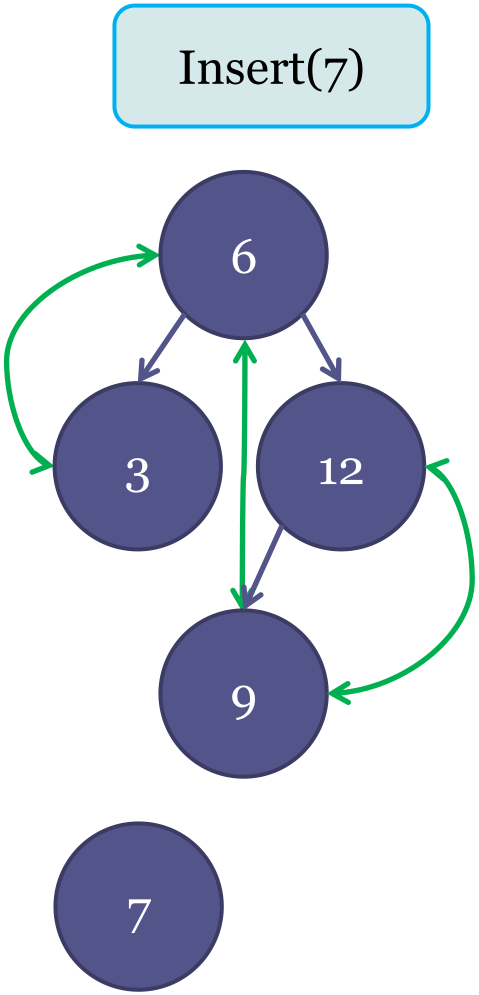

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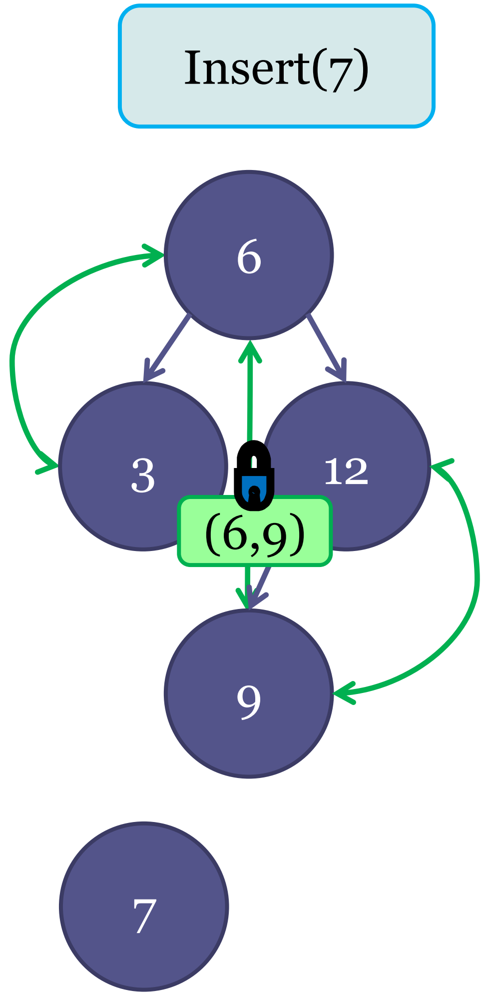
  - If $k \leq l$: lock (l's pred, l)



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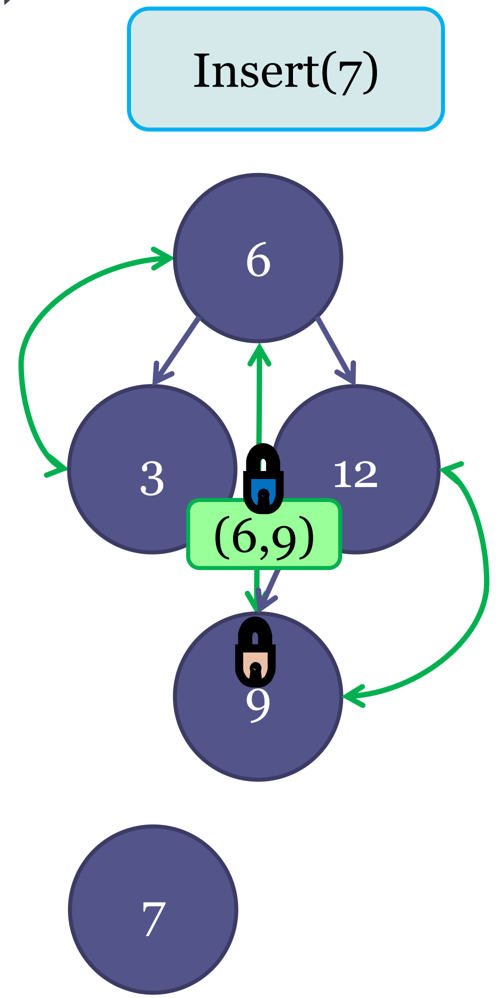

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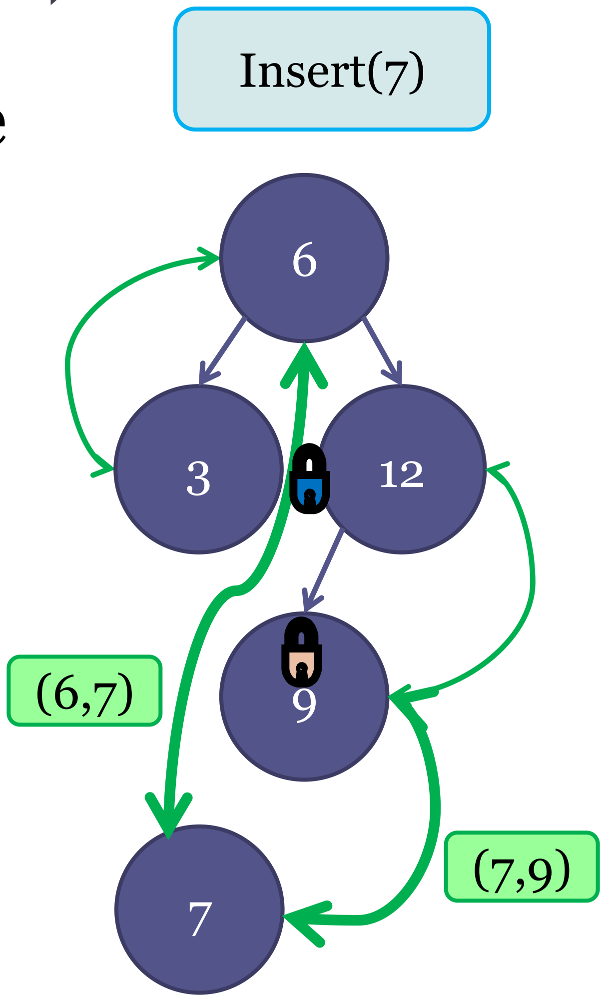

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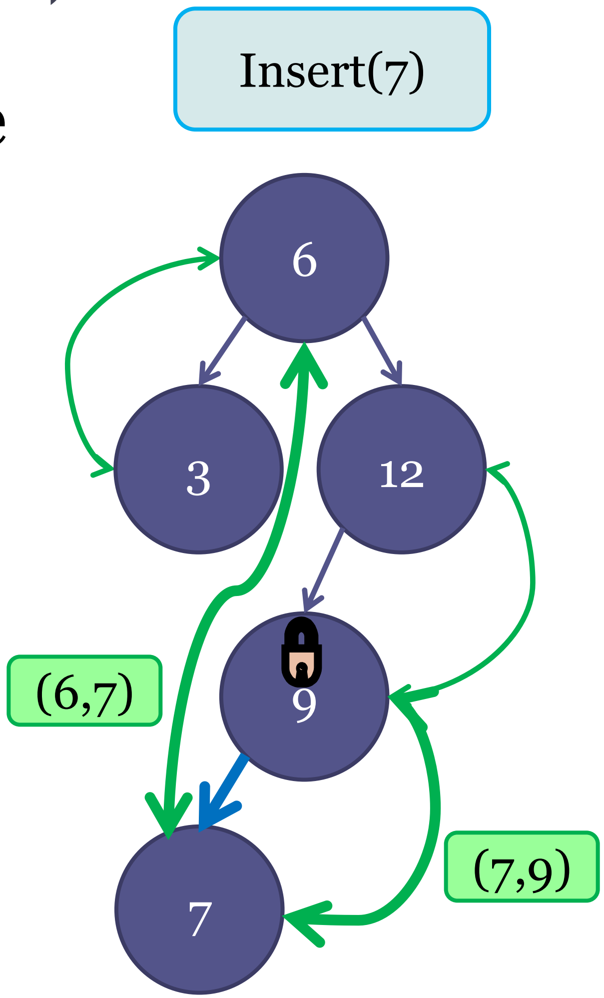

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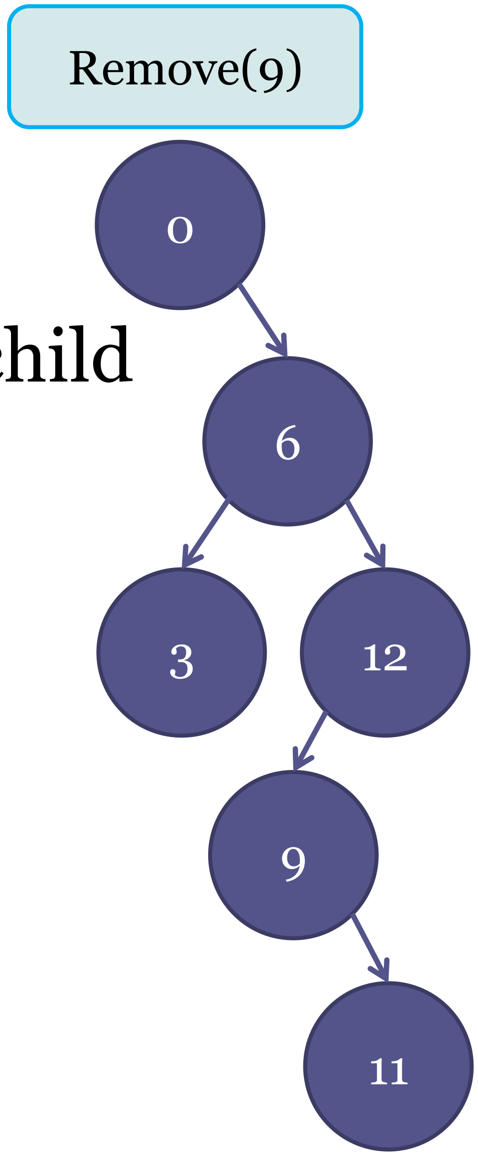

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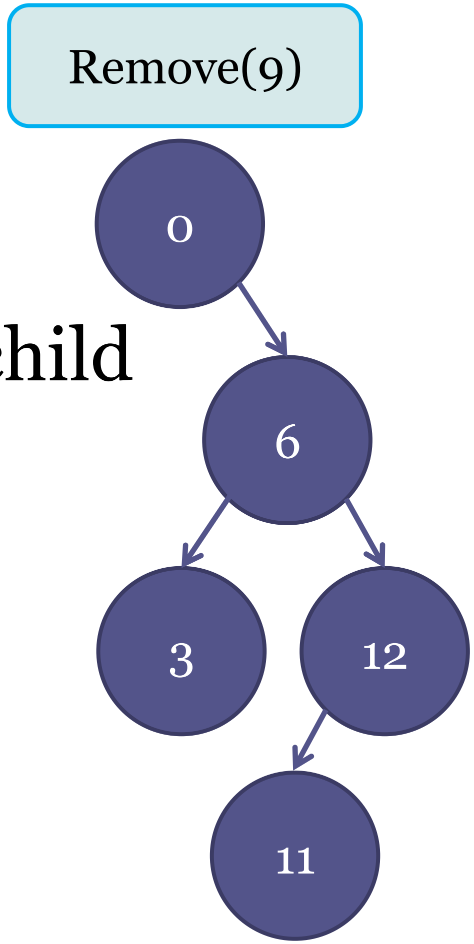

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Remove(6)

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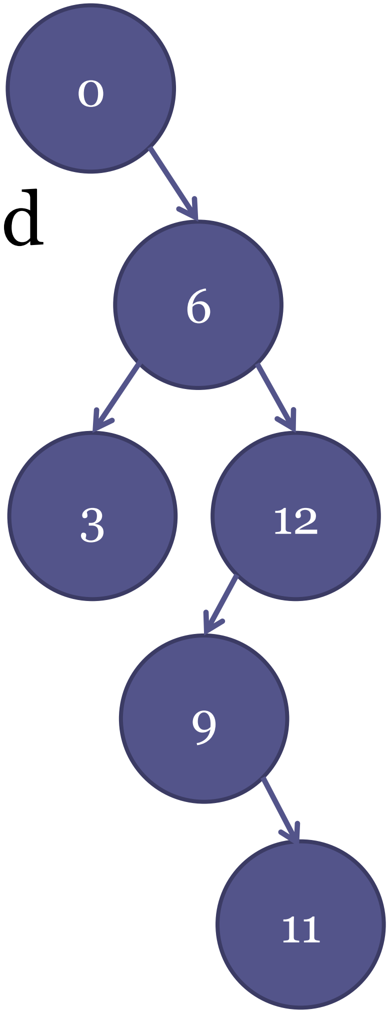
- If the node to remove has 2 children
  - Search for its successor, s
    - The left most node in the right sub-tree
  - Relocate s to its location



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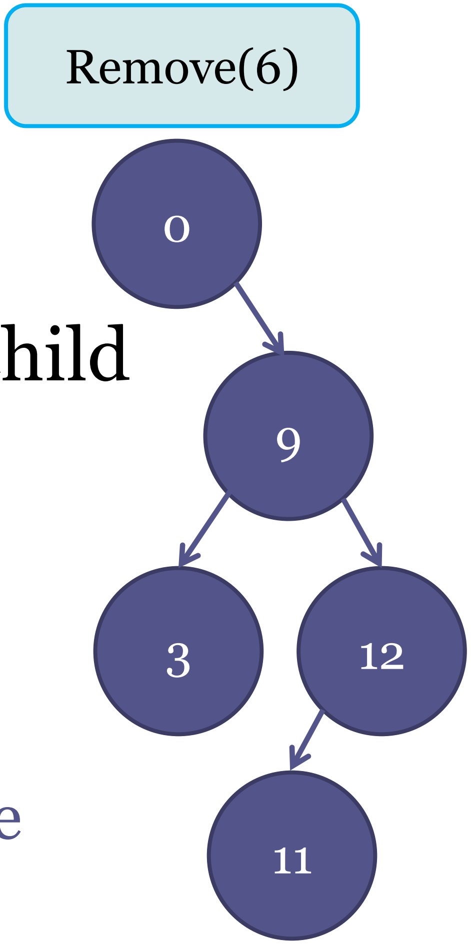

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- Let n be the node to remove
- Lock (n's pred, n)
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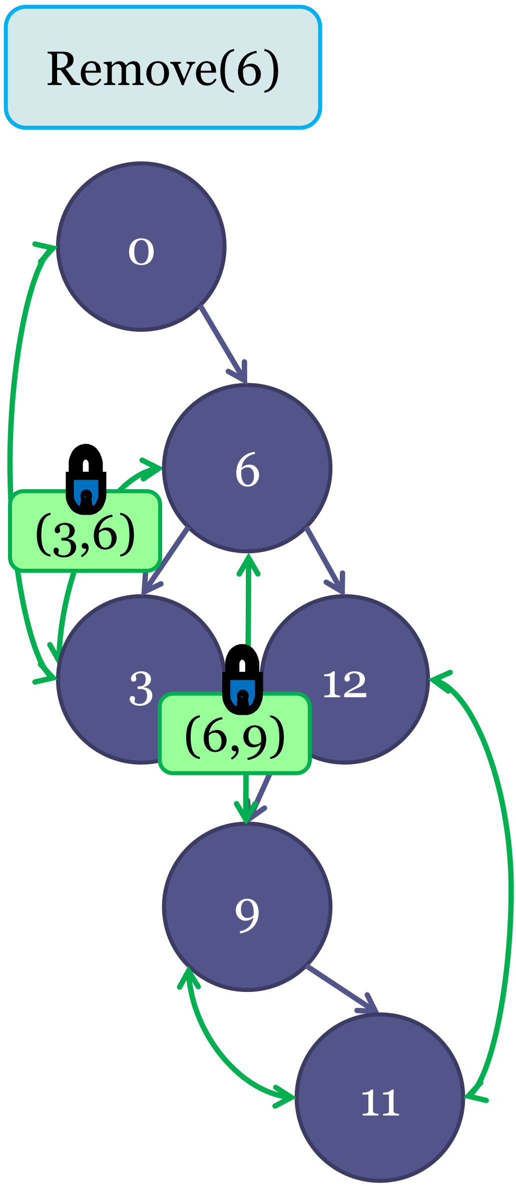

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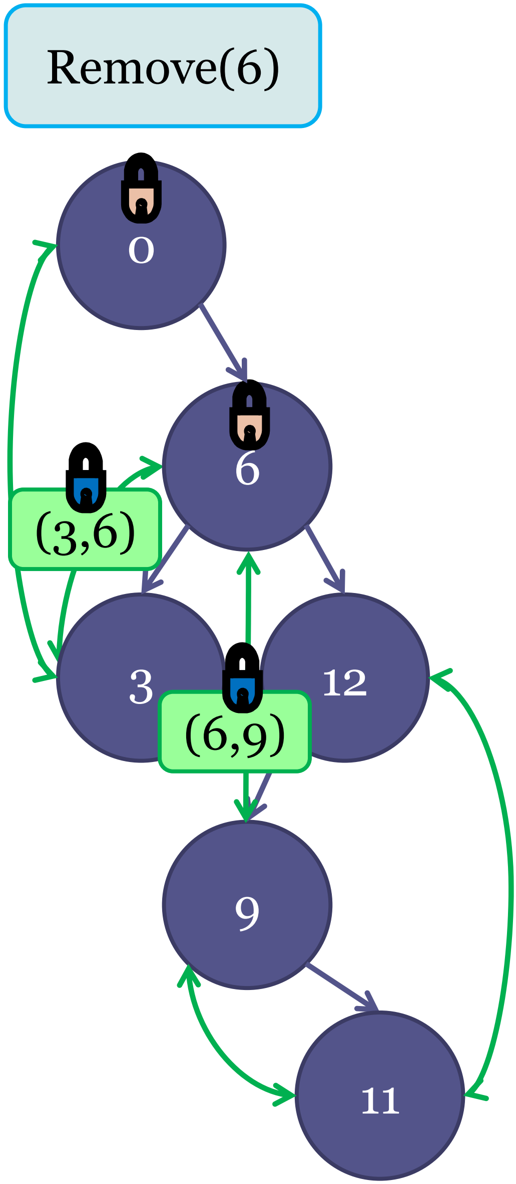
- Lock n and its parent



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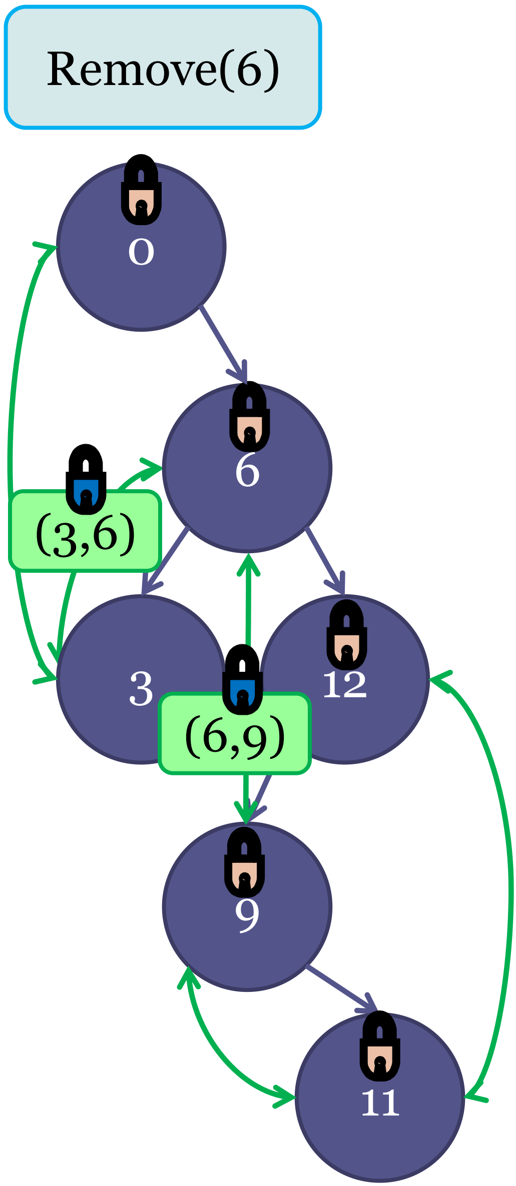

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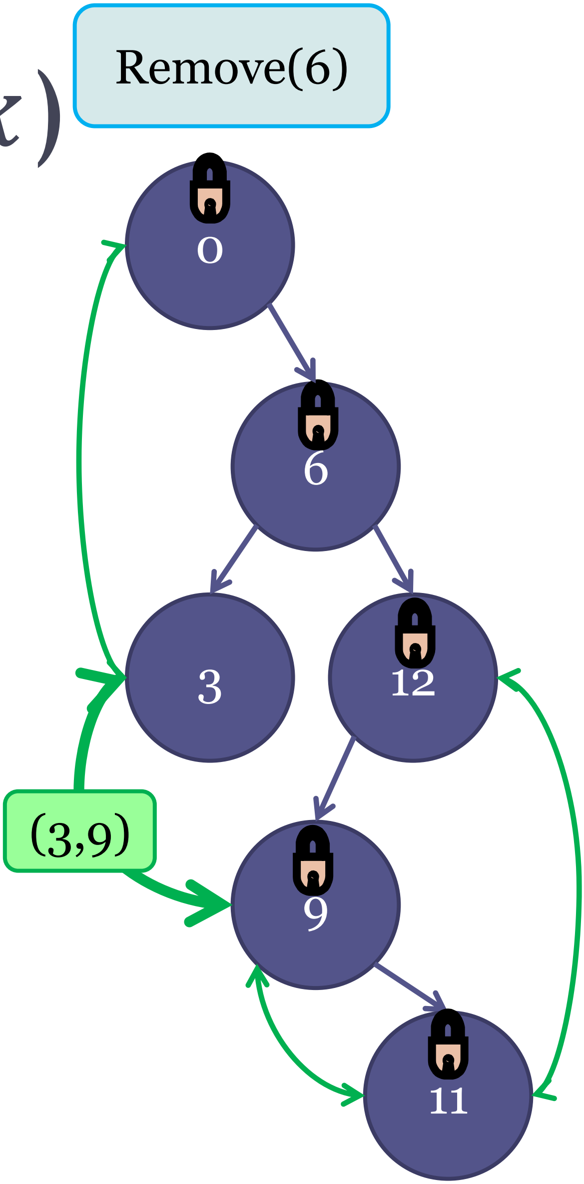

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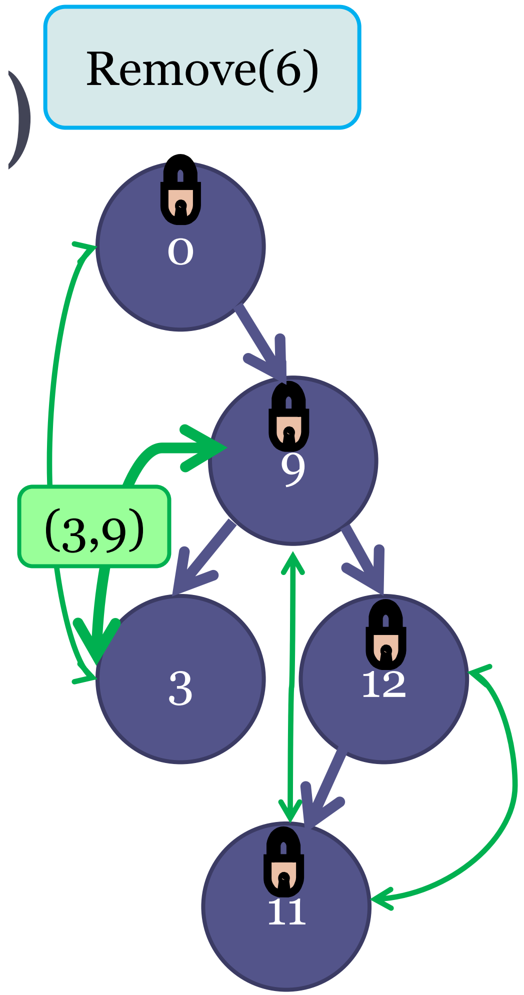

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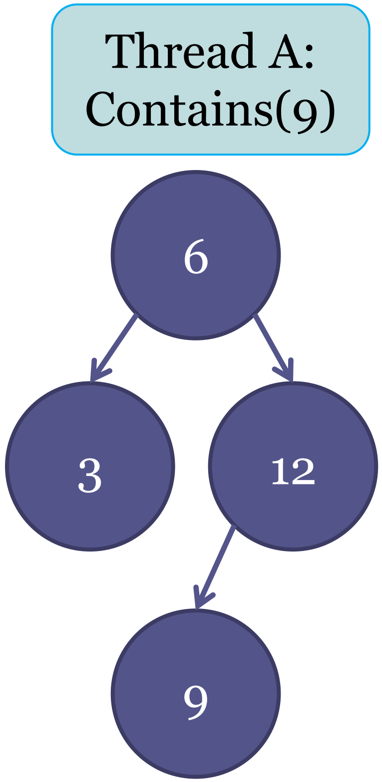

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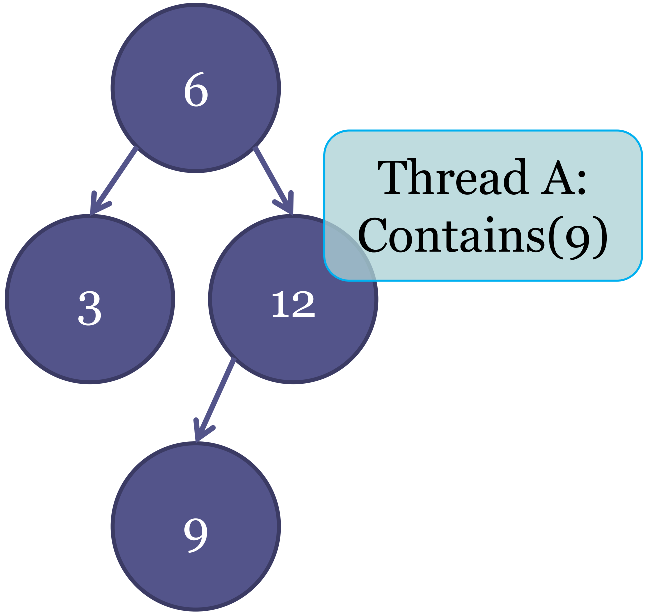

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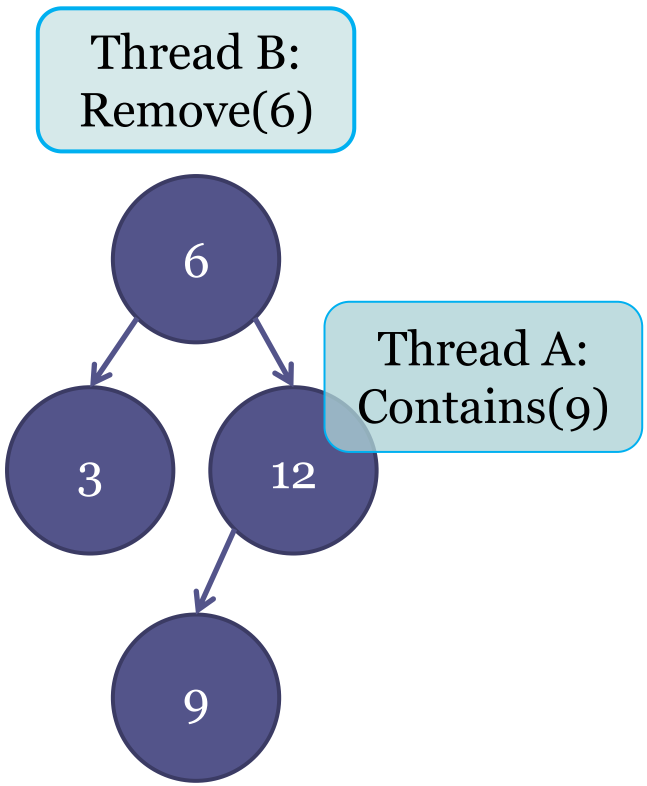

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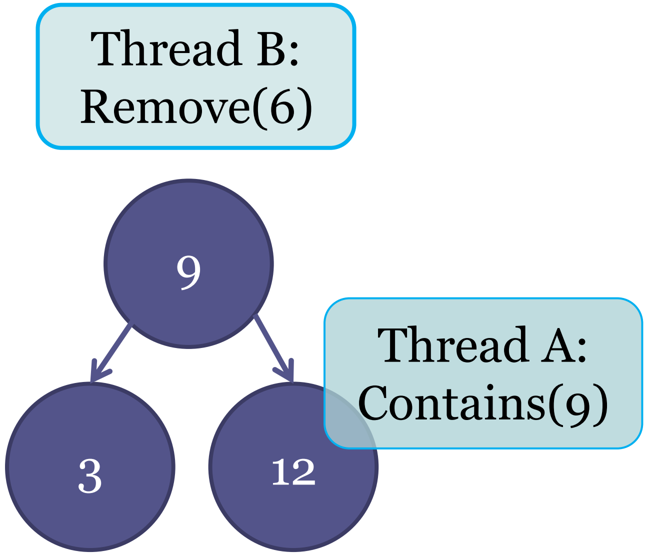

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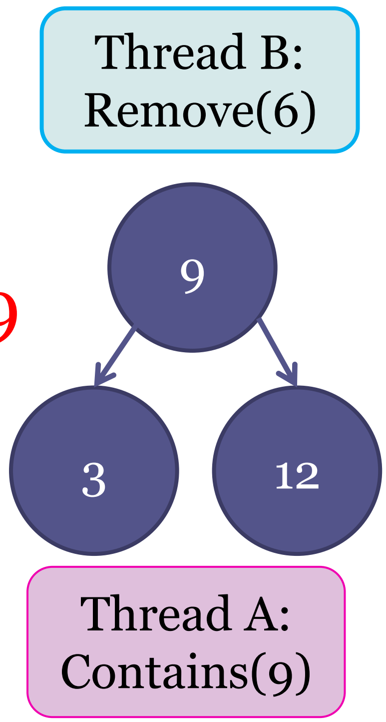

Solution

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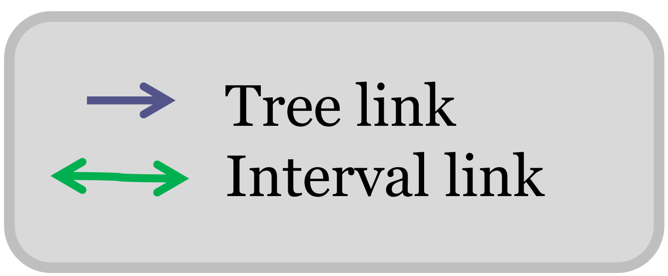

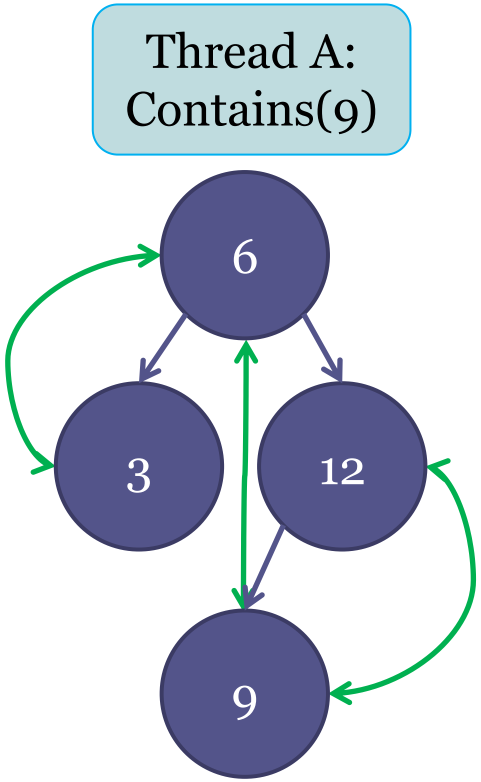

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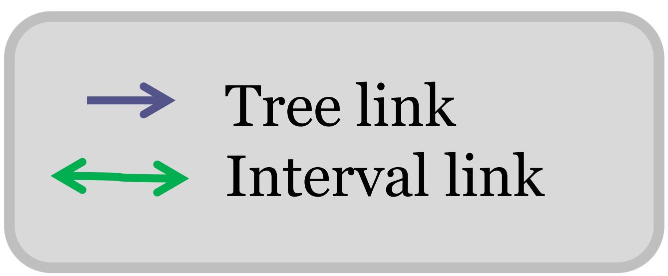
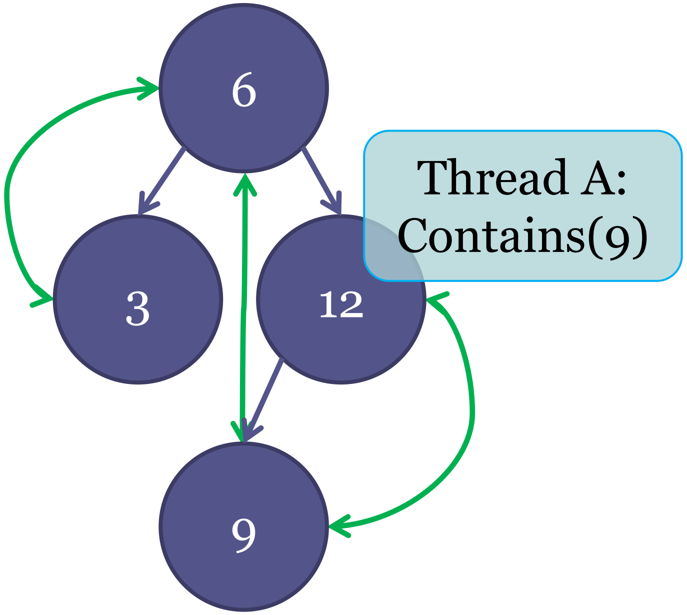
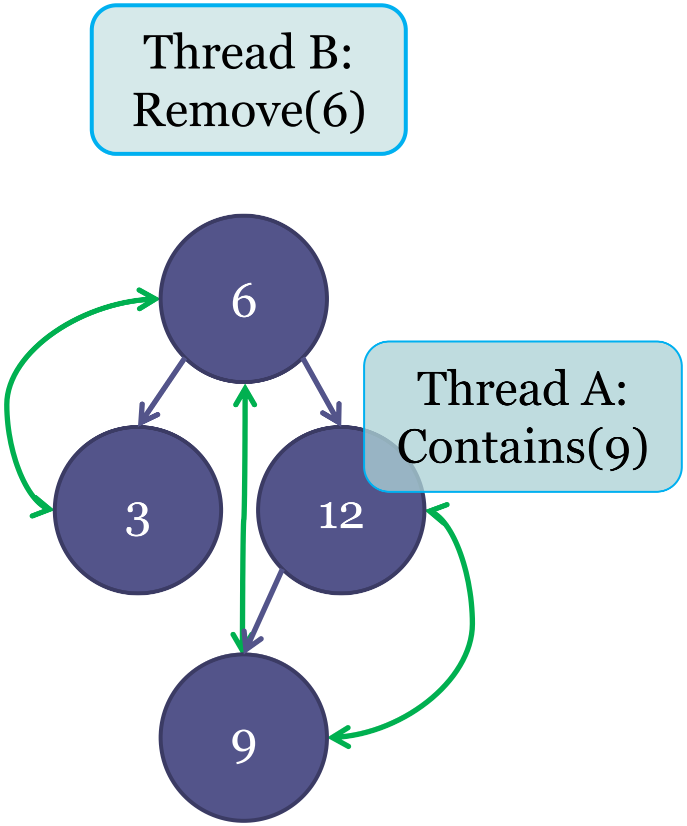




Solution

- Consult the logical ordering layout before making final decisions



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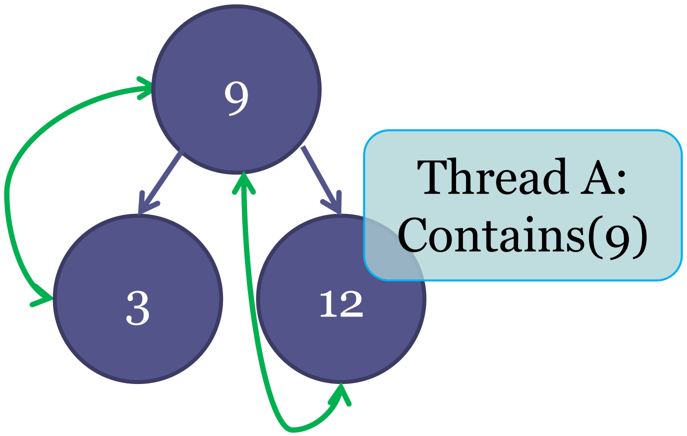

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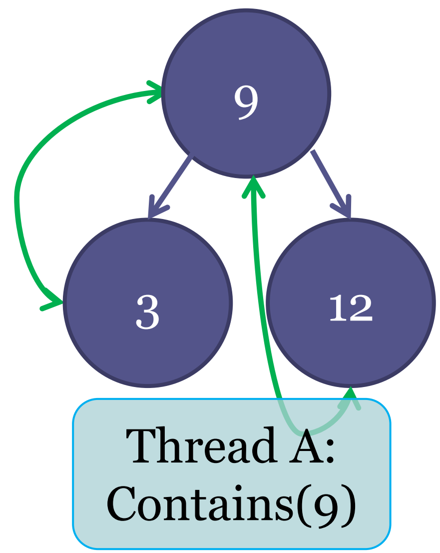
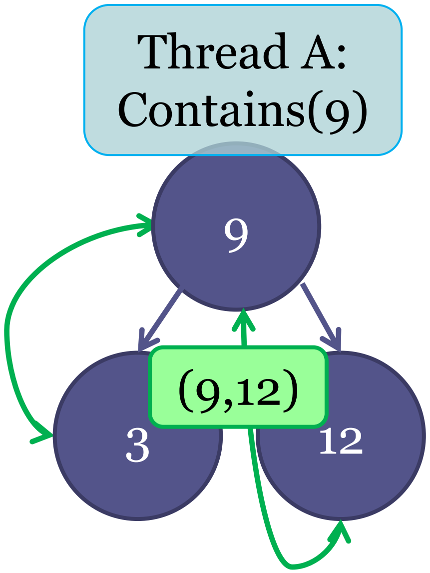


Tree link
Interval link

Solution

- Consult the logical ordering layout before making final decisions



From BST to AVL Tree

- After each update, apply balancing operations
- Balancing operations relocate nodes in the tree
  - Requires only node locks
- Concurrent threads cannot miss keys, since they consult the logical ordering layout

Implementation

- We implemented our BST and AVL tree in Java
- We compared to state-of-the-art algorithms

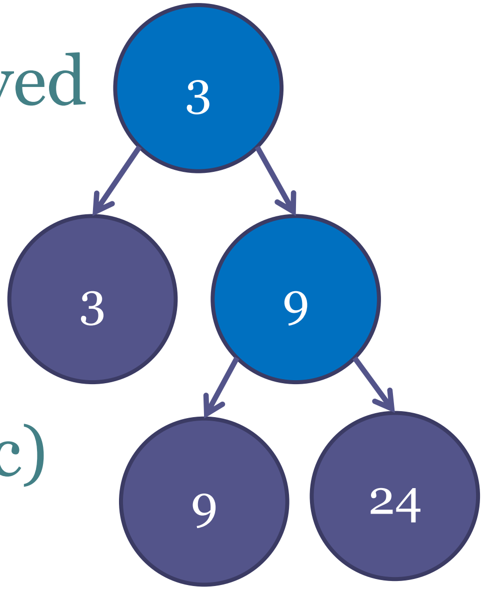
Comparison to Existing Algorithms

- Partially external trees
  - Internal nodes are only marked as removed
    - A follow-up insert can revive them

- Locked-based, partially external trees
  - Bronson et al., PPoPP 2010 (BCCO)
  - A variation of our work (Our LR-AVL)

Comparison to Existing Algorithms

- External tree
  - Elements are kept only in the leaves
  - Inner nodes serve as routing nodes
  - Only leaves can be asked to be removed
  - Traversal paths are typically longer



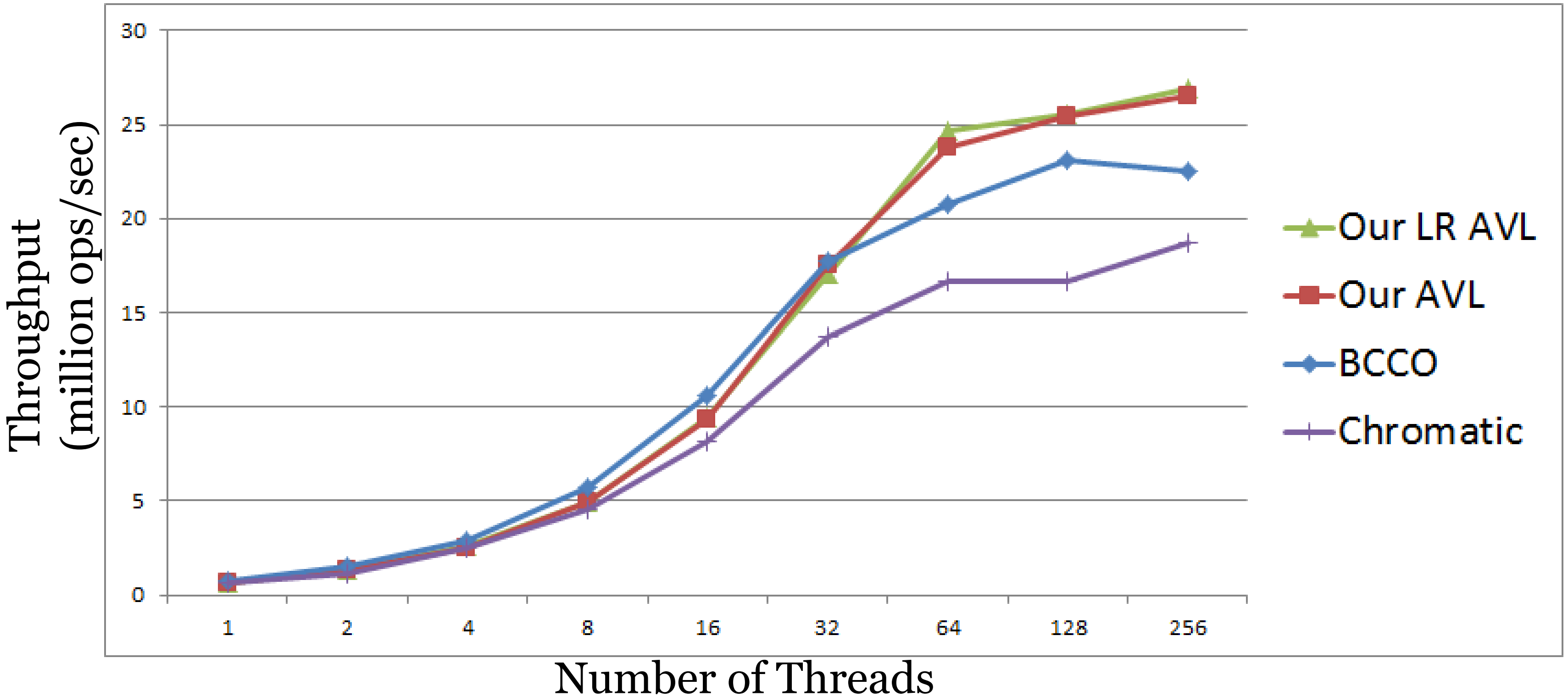
- Non-Blocking external tree
  - Brown et al., PPoPP 2014 (Chromatic)

Evaluation

- A 4-socket AMD Opteron, with 64 h/w threads
- Threads randomly chose operation type and key
  - Different workloads for the operation type
    - 100% contains, 0% insert, 0% remove
    - 70% contains, 20% insert, 10% remove
  - Different key ranges
    - $2 \cdot 10^6, 2 \cdot 10^5$

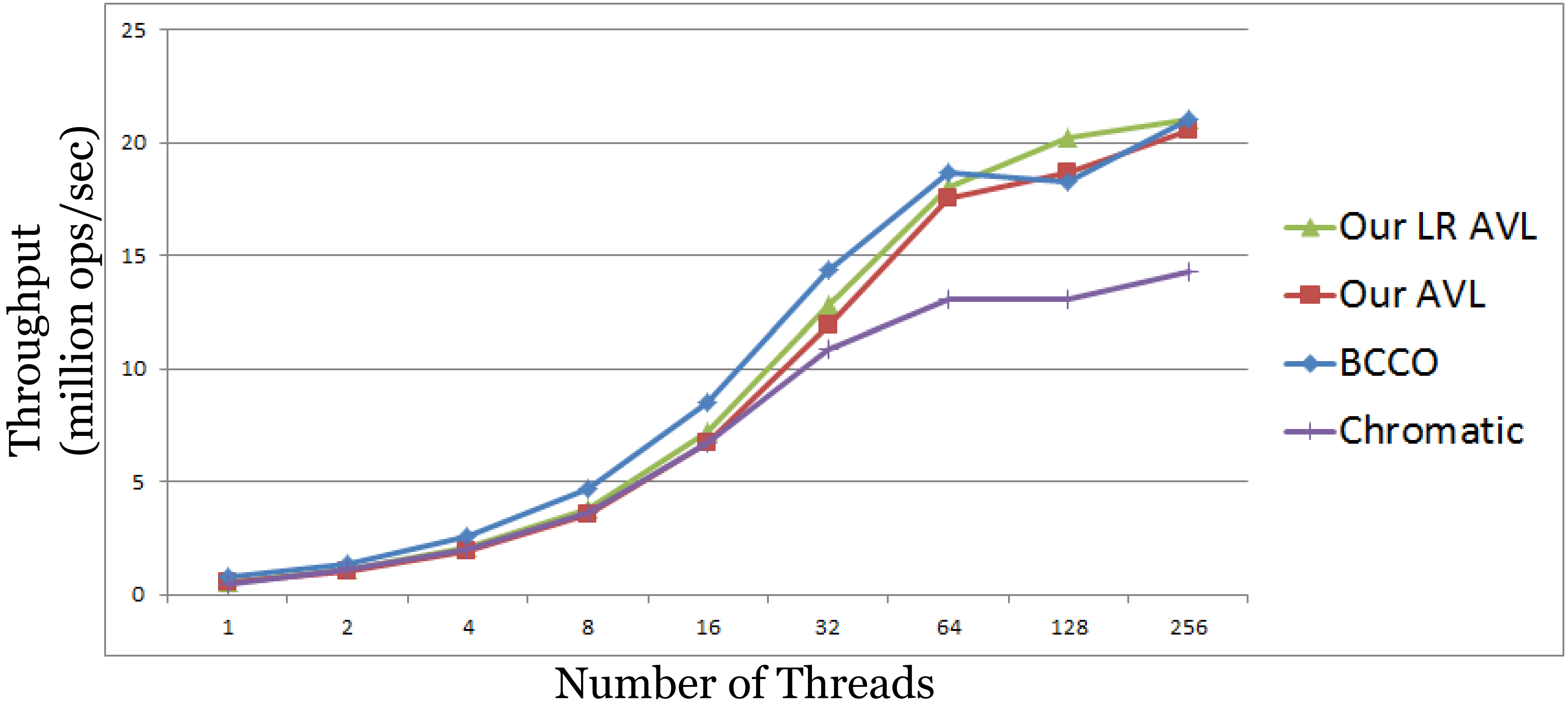
Evaluation

- 100% contains, 0% insert, 0% remove
- Key range: $2 \cdot 10^6$



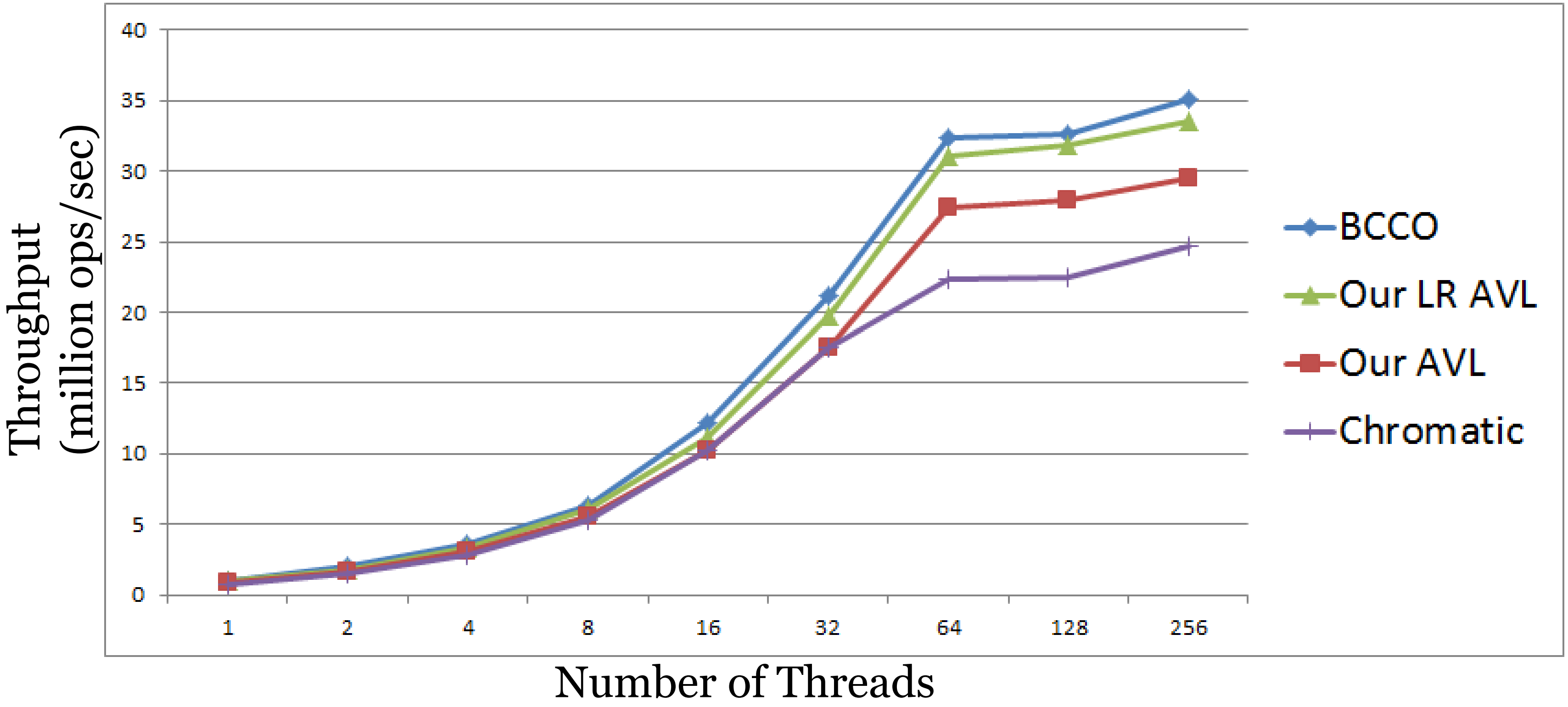
Evaluation

- 70% contains, 20% insert, 10% remove
- Key range: $2 \cdot 10^6$



Evaluation

- 70% contains, 20% insert, 10% remove
- Key range: $2 \cdot 10^5$



Summary

- We presented a new practical concurrent BST
  - Non-blocking search
  - Balanced
  - Efficient
  - Simple
- Our main insight
  - Maintain explicitly the intervals

Thank you!